$Re_j = \omega_j\sigma/\nu$	is the Reynolds number for jet;
ω_j	is the jet velocity at slot;
ν	is the kinematic for bed viscosity;
δ	is the slot width.

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LAWS GOVERNING GAS-BUBBLE MOTION IN A FLUIDIZED BED

Yu. S. Teplitskii and A. I. Tamarin

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Laws governing the motion of particles and gas bubbles in a nonuniform fluidized bed are analyzed on the basis of a variational method for describing the hydrodynamics of a fluidized bed [1] using functions of the potential motion of phases around an individual bubble [2]. Theoretical results are compared with existing experimental data [6-17].

The efficiency of technical processes taking place in a fluidized bed is determined to a considerable extent by the nature of gas-bubble motion.

A large amount of experimental material has been accumulated on the laws governing the motion of individual bubbles artificially injected into a fluidized bed at filtration rates close to the rate for initiation of fluidization [3]. Relations were established which determined the velocity and size of such bubbles. Potential functions were also obtained which described the motion of phases in the neighborhood of a rising gas bubble [2, 4, 5].

There is a large amount of data on the motion of bubbles in a fluidized bed at fluidization numbers greater than one [6-17]. The results of the various investigators are contradictory; the laws governing the motion of bubbles are not clear and there are no sufficiently justified theoretical models which would make it possible to obtain quantitative laws governing the motion of the bubbles.

It was shown [1] that one can obtain a representation of the averaged velocity and phase concentration fields in a nonuniform fluidized bed by using a variational formulation of the motion of a two-phase system.

We consider the following simplified model of a system. We confine ourselves to the two-dimensional case. In accordance with the concepts of the simplest two-phase theory [3], we consider a fluidized bed consisting of an emulsion phase, in which the particle concentration is constant and equal to E_0, and ascending gas bubbles. We arbitrarily divide the bed into cells, each of which consists of a bubble with a following hydrodynamic wake and surrounding emulsion phase. The size of the cell and the radius of the bubble will increase during motion from below upwards. The rate of bubble rise will increase correspondingly. We introduce the quantity n − the number of bubbles at a distance h vertically above the gas-distribution grid.

The velocity fields of the gas and particles, and also the static pressure field outside the bubble and its hydrodynamic wake, are described by known functions [2] which in a fixed coordinate system with an origin coinciding with the center of a rising bubble at a given time are of the form

a) velocity of solid phase:

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$$w_x = u_b - k_1 u_b \left[1 + \frac{D_b^2}{4(x^2+y^2)} - \frac{D_b^2 x^2}{2(x^2+y^2)^2} \right]; \tag{1}$$

$$w_y = \frac{k_1 u_b D_b^2 xy}{2(x^2+y^2)^2}; \tag{2}$$

b) velocity of fluidizing agent

$$v_x = v_{0_1} + u_b - k_1 u_b \left[1 + \frac{D_b^2 (1 + v_{0_1}/k_1 u_b)}{4(x^2+y^2)} \left(1 - \frac{2x^2}{x^2+y^2} \right) \right]; \tag{3}$$

$$v_y = \frac{D_b^2 (v_{0_1} + k_1 u_b) xy}{2(x^2+y^2)^2}; \tag{4}$$

c) static pressure

$$p = p_b - (F_1 - 1) \left[x - \frac{D_b^2 x}{4(x^2+y^2)} \right]. \tag{5}$$

For the region of the hydrodynamic wake of the bubble, we assume

a) velocity of solid phase

$$w_x = u_b; \quad w_y = 0; \tag{6}$$

b) velocity of gas

$$v_x = v_{0_2} + u_b; \quad v_y = 0; \tag{7}$$

c) static pressure

$$\frac{\partial p}{\partial x} = 1 - F_2; \quad \frac{\partial p}{\partial y} = 0. \tag{8}$$

We introduce the quantity $u_b^* = k_1 u_b$ in place of u_b in Eqs. (1)-(4). This circumstance reflects the well-known experimental fact that particle motion in a fluidized bed is circulatory; the sinking motion of solid particles relative to the walls of the apparatus in the space between bubbles is balanced by the rising motion of the material in the hydrodynamic wakes of the bubbles. In Eqs. (5) and (8), note that $1 - F_1$ and $1 - F_2$ are averaged drops in static pressure along the vertical for the corresponding regions. We assume the gas pressure within the bubble (p_b) is constant.

We write the functional of [1] for this cellular model, having modified it somewhat:

$$L = \int_0^{\hat{p}-1} \int_{S_\text{я}} n^0 u_b^0 \Big[-\text{Eu}(1-\text{E}_0) \frac{\partial p}{\partial x_i} v_i^0 - \text{Eu}\,\text{E}_0 \frac{\partial p}{\partial x_i} w_i^0 + \frac{1}{\text{Fr}} \text{E}_0 w_x +$$

$$+ \text{Eu} \frac{\partial p^0}{\partial x_i} v_i + \text{E}_0 w_j^0 w_i \frac{\partial w_i^0}{\partial x_j} + K f(\text{E}_0)(v_i^0 - w_i^0)(v_i - w_i) + \text{E}_0 w_i \left(\pm \frac{\partial w_i^0}{\partial t} - \frac{\partial w_i^0}{\partial x} u_b^0 \right) \Big] dS dt, \tag{9}$$

the minus sign for $\partial w_i^0/\partial t$ refers to the time derivative of the descending circulatory component of w_x from Eq. (1), which is $(1 - k_1)u_b$ since $dt = -dh/u_b$ for this derivative. All quantities are dimensionless in Eqs. (1)-(9) and in the following, and primes are omitted for convenience.

By considering the form of the functional (9) and the condition for retention of particles in the system, it is easy to show that L is the power the gas expends during filtration through the bed.

We substitute the system of trial functions (1)-(8) into Eq. (9).

After integration over the cell volume, we obtain

$$L = \int_0^{\hat{p}-1} n^0 u_b^0 \Big\{ \text{Eu}(1-\text{E}_0)(F_1 - 1)[v_{0_1}^0 + (1-k_1)u_b^0] + \text{Eu}\,\text{E}_0 (F_1 - 1) \times$$

$$\times (1-k_1)\, u_b^0 + \frac{1}{\mathrm{Fr}}\, \mathrm{E}_0 (1-k_1)\, u_b - \mathrm{Eu}\,(F_i^0 - 1)[v_{0_1} + (1-k_1)\, u_b] +$$

$$+ Kf(\mathrm{E}_0)\, v_{0_1}^0 v_{0_1} - (1-k_1)^2\, \mathrm{E}_0\, u_b \frac{du_b^0}{dt} \Big\} \frac{\pi}{4} (b^2 - D_b^2)\, dt +$$

$$+ \int_0^{\hat{p}-1} n^0 u_b^0 \{\mathrm{Eu}\,(1-\mathrm{E}_0)(F_1 - 1)[v_{0_1}^0 + k_1 u_b^0] + \mathrm{Eu}\,\mathrm{E}_0 (F_1 - 1)\, k_1 u_b^0 -$$

$$- \mathrm{Eu}\,(F_1^0 - 1)(v_{0_1} + k_1 u_b) + Kf(\mathrm{E}_0)\, v_{0_1}^0 v_{0_1} +$$

$$+ k_1^2\, \mathrm{E}_0\, u_b \frac{du_b^0}{dt} \Big\} \frac{\pi}{4} D_b^2 \left[1 - \left(\frac{D_b}{b}\right)^2 \right] dt + \int_0^{\hat{p}-1} n^0 u_b^0 \Big[\mathrm{Eu}\,(1-\mathrm{E}_0)(F_2 - 1) \times$$

$$\times (v_{0_2}^0 + u_b^0) + \mathrm{Eu}\,\mathrm{E}_0 (F_2 - 1)\, u_b^0 + \frac{1}{\mathrm{Fr}}\, \mathrm{E}_0 u_b - \mathrm{Eu}\,(F_2^0 - 1)(v_{0_2} + u_b) +$$

$$+ Kf(\mathrm{E}_0)\, v_{0_2}^0 v_{0_2} + \mathrm{E}_0\, u_b \frac{du_b^0}{dt} \Big] \frac{\pi}{4} \frac{\alpha}{1+\alpha} D_b^2\, dt. \qquad \textbf{(10)}$$

By varying L in the form (10) over v_{0_1}, v_{0_2}, and u_b, we obtain the following relations (after variation, quantities with and without the superscript 0 are equated to one another [1]):

$$\mathrm{Eu}\,(F_1 - 1) = Kf(\mathrm{E}_0)\, v_{0_1} \qquad \textbf{(11)}$$

(variation over v_{0_1}),

$$\mathrm{Eu}\,(F_2 - 1) = Kf(\mathrm{E}_0)\, v_{0_2} \qquad \textbf{(12)}$$

(variation over v_{0_2}),

$$\left[\frac{1}{\mathrm{Fr}}\, \mathrm{E}_0 - \mathrm{Eu}\,(F_1 - 1) - (1-k_1)\, \mathrm{E}_0 \frac{du_b}{dt}\right](1-k_1) \frac{\pi}{4} (b^2 - D_b^2) +$$

$$+ \left[-\mathrm{Eu}\,(F_1 - 1) + \mathrm{E}_0 k_1 \frac{du_b}{dt}\right] k_1 \frac{\pi}{4} D_b^2 \left[1 - \left(\frac{D_b}{b}\right)^2 \right] = 0 \qquad \textbf{(13)}$$

(variation over u_b – continuous phase),

$$\frac{1}{\mathrm{Fr}}\, \mathrm{E}_0 - \mathrm{Eu}\,(F_2 - 1) + \mathrm{E}_0 \frac{du_b}{dt} = 0 \qquad \textbf{(14)}$$

(variation over u_b - region of bubble wake); Eqs. (11) and (12) represent equations of balance for gas momentum in the corresponding regions in terms of the trial functions (1)-(8). Equations (13) and (14) are the analogous form of the sum of the equations of balance for the momenta of solid particles and gas.

A logical consequence of two-phase theory is the following equality,

$$\int_0^{\hat{p}-1} nu_b\, \mathrm{Eu}\,(1-\mathrm{E}_0)(F_1 - 1)\, v_{0_1} \frac{\pi}{4} (b^2 - D_b^2)\, dt + \int_0^{\hat{p}-1} nu_b\, \mathrm{Eu}\,(1-\mathrm{E}_0) \times$$

$$\times (F_1 - 1)\, v_{0_1} \frac{\pi}{4} D_b^2 \left[1 - \left(\frac{D_b}{b}\right)^2\right] dt + \int_0^{\hat{p}-1} nu_b\, \mathrm{Eu}\,(1-\mathrm{E}_0)(F_2 - 1) \times$$

$$\times v_{0_2} \frac{\pi}{4} D_b^2 \frac{\alpha}{1+\alpha}\, dt = \int_0^{\hat{p}-1} nu_b\, \mathrm{Eu}\,(1-\mathrm{E}_0)(F - 1)\, v_0 \frac{\pi}{4} \left(b^2 - D_b^2 \frac{1}{1+\alpha}\right) dt = L_0, \qquad \textbf{(15)}$$

since in accordance with this theory, the relative velocity of the phases remains on the average equal to v_0 as it was at the beginning of fluidization; L_0 is the power which the gas expends during filtration through the bed at the beginning of fluidization.

Using Eqs. (11)-(14) and Eq. (15), the functional (10) takes the form:

$$L = L_0 + \int_0^{\hat{p}-1} nu_b \,\mathrm{Eu}\,(F_1 - 1)(1 - k_1)\, u_b \frac{\pi}{4}\left(b^2 - D_b^2\right) dt +$$

$$+ \int_0^{\hat{p}-1} nu_b \,\mathrm{Eu}\,(F_1 - 1)\, k_1 u_b \; \frac{\pi}{4} D_b^2 \left[1 - \left(\frac{D_b}{b}\right)^2\right] dt + \int_0^{\hat{p}-1} nu_b \,\mathrm{Eu}\,(F_2 - 1)\, u_b \; \frac{\pi}{4} \cdot \frac{\alpha}{1+\alpha} D_b^2\, dt. \quad \textbf{(16)}$$

Substituing in Eq. (16) the expressions for $F_1 - 1$ and $F_2 - 1$ from Eqs. (13) and (14), we obtain

$$L = L_0 + \int_0^{\hat{p}-1} nu_b \Big\{ (1 - k_1)(b^2 - D_b^2) \frac{1}{\mathrm{Fr}} \mathrm{E}_0 - (1 - k_1)^2 (b^2 - D_b^2)\, \mathrm{E}_0 \times$$

$$\times \frac{du_b}{dt} + k_1^2 D_b^2 \left[1 - \left(\frac{D_b}{b}\right)^2\right] \mathrm{E}_0 \frac{du_b}{dt} \Big\} \frac{\pi}{4}\, u_b dt + \int_0^{\hat{p}-1} nu_b \left[\mathrm{E}_0 \frac{1}{\mathrm{Fr}}\, u_b + \mathrm{E}_0 u_b \frac{du_b}{dt}\right] \frac{\pi}{4} \frac{\alpha}{1+\alpha} D_b^2\, dt. \quad \textbf{(17)}$$

At the beginning of fluidization $(u = u_0) k_1 = 1$ and $D_b = 0$, and the natural conclusion $L = L_0$ follows from Eq. (17).

The physical meaning of Eq. (17) is the following: The excess power of the gas $(L - L_0)$ is expended in acceleration of the particles and in an increase of their potential energy in the gravitational field within the hydrodynamic wakes of the bubbles, and in **acceleration of the apparent mass of a bubble** $(\rho_p k_1^2 E_0 (\pi/4) D_b^2 [1 - (D_b/b)^2])$. The latter results from acceleration of particles around a bubble because of the increase in its velocity. In the continuous phase, there additionally occurs conversion of the potential energy of the particles and of the kinetic energy of their descending circulatory motion into potential energy of the gas.

Using the equality

$$\int_0^{\hat{p}-1} nu_b \left[\frac{1}{\mathrm{Fr}} \mathrm{E}_0 (1 - k_1)\, u_b\right] \frac{\pi}{4} (b^2 - D_b^2)\, dt + \int_0^{\hat{p}-1} nu_b \frac{1}{\mathrm{Fr}}\, \mathrm{E}_0 u_b \frac{\pi}{4} \frac{\alpha}{1+\alpha} D_b^2\, dt = 0, \quad (18)$$

which reflects the equality of the circulatory flows in the system, we obtain from Eq. (17)

$$L = L_0 + \int_0^{\hat{p}-1} nu_b \left[(2 - k_1)(k_1 - 1)\, \mathrm{E}_0 u_b \frac{du_b}{dt}\right] \frac{\pi}{4} (b^2 - D_b^2)\, dt + \int_0^{\hat{p}-1} nu_b k_1^2\, \mathrm{E}_0 u_b \frac{du_b}{dt} \frac{\pi}{4} D_b^2 \left[1 - \left(\frac{D_b}{b}\right)^2\right] dt. \quad (19)$$

Assuming

$$L = \int_0^{\hat{p}-1} nu_b \,\mathrm{Eu}\,(F_{av} - 1)\, u \frac{\pi}{4} b^2 dt, \quad (20)$$

where $F_{av} - 1 = \Delta p/\hat{p}$, and using the equality

$$(F_{av} - 1)\, b^2 = (F - 1)\left(b^2 - D_b^2 \frac{1}{1+\alpha}\right), \quad (21)$$

we obtain from Eq. (19)

$$\mathrm{Eu}\,(F_{av} - 1)(u - u_0) = \mathrm{E}_0 u_b \frac{du_b}{dt} \; \{(2 - k_1)(k_1 - 1)[1 - \mathrm{E}_b(1+\alpha)] + k_1^2\, \mathrm{E}_b (1+\alpha)[1 - \mathrm{E}_b(1+\alpha)]\}, \quad (22)$$

where $[1/(1+\alpha)](D_b/b)^2 = E_b$ is the concentration of bubbles in the system.

In dimensional form, Eq. (22) will be

$$g(u - u_0) = u_b \frac{du_b}{dt} \hat{p} \{(2 - k_1)(k_1 - 1)[1 - \mathrm{E}_b(1+\alpha)] + k_1^2\, \mathrm{E}_b(1+\alpha)[1 - \mathrm{E}_b(1+\alpha)]\}, \quad (23)$$

assuming $\Delta p/H_0 = \rho_p g E_0$ in Eq. (22).

Relations such as Eq. (18) are obviously valid for each cell so that one can write

$$(k_1 - 1)[1 - (1+\alpha)\, \mathrm{E}_b] = \alpha \mathrm{E}_b. \quad (24)$$

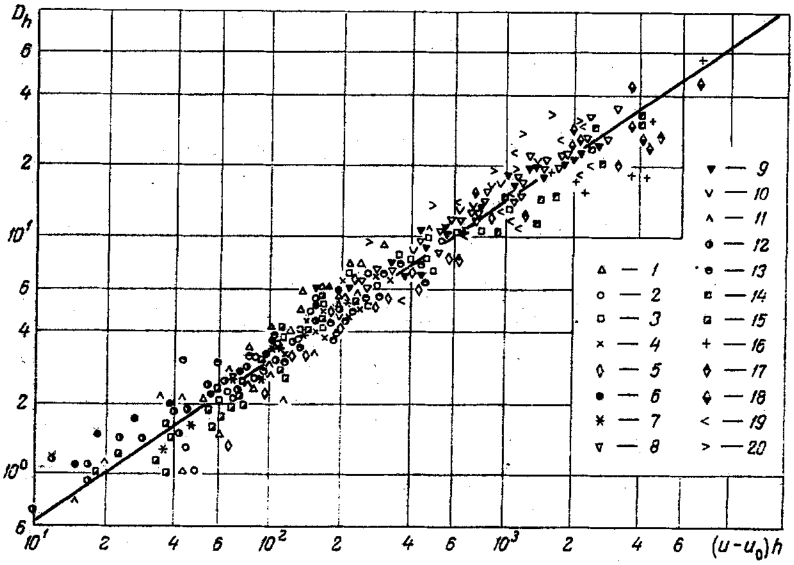

Fig. 1. Vertical bubble size D_h, cm, as a function of $(u-u_0)h$, cm²/sec: 1-5) [6] (for particles of aluminum, carbon, quartz, glass beads, and powdered glass, respectively); 6) [9]; 7) [10]; 8, 9) [11] (61 × 61 cm and 122 × 122 cm columns, respectively); 10) [12]; 11) [13]; 12) [14] (rounded particles); 13) [14] (particles of irregular shape); 14) [8]; 15) [15]; 16) [16] (values calculated from experiments on expansion of bed); 17) [17]; 18) [16] (values calculated from experiments on mass transfer); 19, 20) [7] (sand and silica gel, respectively).

Using Eq. (24), Eq. (23) takes the form

$$g(u-u_0) = \Psi(\alpha, E_b)\, u_b \frac{du_b}{dt}. \tag{25}$$

Neglecting terms such as αE_b, $(\alpha E_b)^2$, and $E_b(E_b - \alpha)$ in $\Psi(\alpha, E_b) = ([2 - k_1]\alpha E_b + k_1^2 E_b[1+\alpha][1 - E_b(1+\alpha)])\hat{p}$ and assuming $k_1 \cong 1$, we obtain the approximate evaluation

$$\Psi(\alpha, E_b) \cong E_b \hat{p} = (\hat{p} - 1), \tag{26}$$

for $\Psi(\alpha, E_b)$, where it was assumed $\alpha = 0.2$-0.4 [3], $E_b = 0.1$-0.2, and E_b constant over the height of the bed.

By considering developed fluidization modes where bed expansion is stabilized, it is possible to consider the coefficient $\Psi(\alpha, E_b)$ independent of $(u - u_0)$ and to assume it is a constant: $\Psi(\alpha, E_b) = C$.

By expressing the relation between dh and dt, $dt = dh/u_b$, we obtain from Eq. (25)

$$\int_{u-u_0}^{u_b} u_b^2\, du_b = \frac{1}{C} g(u-u_0) \int_0^h dh. \tag{27}$$

when the remarks made above are taken into consideration. After integration we have

$$v_b^3 \left[3\left(\frac{u-u_0}{v_b}\right)^2 + 3\,\frac{u-u_0}{v_b} + 1 \right] = \frac{3}{C} g(u-u_0)h. \tag{28}$$

At high filtration rates where the quantity $\hat{p} - 1$ is independent of $(u - u_0)$, the ratio $(u - u_0)/v_b$ can also be considered independent of $(u - u_0)$. We then obtain from Eq. (28),

$$v_b \cong q_0 [(u-u_0)gh]^{1/3}. \tag{29}$$

We use the well-known relation between the relative rate of bubble rise and bubble diameter [3]

$$v_b = k_2 \sqrt{gD_b}. \tag{30}$$

Using Eq. (29), we obtain from Eq. (30)

$$D_b = \frac{q}{\sqrt[3]{g}} [(u - u_0)h]^{2/3}. \tag{31}$$

For comparison of the various published data on gas-bubble sizes, all information about their sizes was reduced by us to a single quantity – the vertical dimension of a bubble (D_h) assuming a relation between D_h and D_b in the form $D_h = 0.7D_b$ [14].

Figure 1 shows data from [6-17] treated in accordance with Eq. (31). As is clear from the figure, all experimental points are generalized by a straight line with a slope of 2/3 in the selected coordinate system. The equation for this straight line is

$$D_h = D_0 + \frac{1.3}{\sqrt[3]{g}} [(u - u_0)h]^{2/3}. \tag{32}$$

The relative standard deviation of the experimental points about Eq. (32) is 20%. Note that $D_0 \cong 0$ for all data except for results from [11] in which gas-bubble sizes were studied in columns with bubble-cap gas distributors; in that case, $D_0 = 4$ cm.

Thus, the relation (31) found is well confirmed by numerous experimental data. Equation (32) can be used for calculating the size of gas bubbles in commercial equipment with a fluidized bed within the following parameter ranges: $10 \le D_k \le 180$ cm; $0.5 \le u_0 \le 8$ cm/sec; $40 \le (u - u_0)h \le 7000$ cm^2/sec (D_k is the diameter of the equipment).

NOTATION

b	is the cell diameter;
D_b, D_h	are the diameter and vertical dimension of bubble;
D_0	is the initial diameter of bubble;
H_0, H	are the height of bed at filtration rates u_0 and u;
h	is the above gas-distribution grid;
g	is the acceleration of free fall;
k	is the coefficient of friction;
$H/H_0 = \hat{p}$	is the expansion of bed;
p_b	is the pressure in bubble;
p_0, p	are the atmospheric and static pressure;
Δp	is the pressure drop in bed;
k_1, k_2, q_0, q	are the dimensionless coefficients;
S_c	is the cell area;
u_b, v_b	are the absolute and relative rates of bubble rise;
u_b^*	$= k_1 u_b$;
u_0, u	are the initial rate of fluidization and filtration rate;
v_i, w_i	are the gas and particle velocities;
$v_0 = u_0/(1 - E_0)$	are the velocity in gaps between particles;
t	is the time;
T_0	$= H_0/(u - u_0)$;
x, y	are the coordinates;
$D_b/H_0 = D_b'$; $b/H_0 = b'$, $x/H_0 = x'$; $y/H_0 = y'$; $u_b/(u - u_0) = u_b'$; $v_0/(u - u_0) = v_0'$; $u_b^*/(u - u_0) = u_b^{*\prime}$; $u/(u - u_0) = u'$; $u_0/(u - u_0) = u_0'$; $p/p_0 = p'$; $\Delta p/p_0 = \Delta p'$; $t/T_0 = t'$	are all dimensionless quantities;
$F - 1$	$= \Delta p'$;
$Eu = p_0/\rho_p(u - u_0)^2$	is the Euler number;
$Fr = (u - u_0)^2/gH_0$	is the Froude number;
$K = kH_0/\rho_p(u - u_0)$	is the dimensionless coefficient of friction;
α	is the fractional volume of hydrodynamic wake of bubble (fraction of bubble volume);
E_0, E	is the particle concentration in continuous phase at u_0 and u;
ρ_p	is the particle density.

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OPTIMAL HORIZONTAL PNEUMATIC TRANSPORT

V. V. Kazantsev and M. B. Rivkin

UDC 621.867.8

A study has been made of the effects of periodic large-scale pressure perturbations on the passage of air through a moving bed of granular material in dense-phase horizontal pneumatic transport.

Experiment shows [1] that horizontal pneumatic transport allows the throughput to be increased by increasing the air speed up to some limit, after which an adverse effect sets in, which cannot be explained in terms of existing views on the mechanism of motion in high-concentration two-phase mixtures, according to which the mixing in the lower layer of material (bed) occurs on account of the tangential stresses proportional to the air speed acting at the phase interface [2]. Also, this model fails to explain the very considerable pressure fluctuations accompanying the motion of the mixture through the pipeline (Fig. 1).

The studies on the structure of high-concentration flows [3] provide the following model for the transport; most of the material is transported in the lower part of the pipeline at a constant porosity m_0 as a bed whose height and structure vary little along the length [2]. Ridges or dunes travel along the upper surface of the bed [4], and the air flowing over these gives rise to periodic pressure perturbations, which interact with the air flowing through the bed. This in turn gives rise to an oscillating force within the bed, which is directed along the line of flow and tends to accelerate the bed.

As the frequency of ridge passage is a single-valued function of the air speed, we have to examine the transient-state passage of the air through the bed for a fixed porosity in response to two forces: a constant pressure gradient and a periodic pressure perturbation at the upper boundary.

The following is [5] the linearized equation for isothermal infiltration:

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial z^2} - M\frac{\partial P}{\partial t} = 0, \tag{1}$$

$$0 < x < L, \quad 0 < z < H, \quad t > 0, \quad M = \varepsilon m_0/kp_0, \quad P = p^2.$$

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